

**APPROVED  
at a meeting of the  
Scientific Council  
NJSC «Al-Farabi KazNU».  
Minutes No.10 dated  
May 13, 2023.**

**The program of the entrance exam for applicants to the PhD  
for the group of educational programs  
D093 - «Mechanics»**

**General provisions.**

1. The program was drawn up in accordance with the Order of the Minister of Education and Science of the Republic of Kazakhstan dated October 31, 2018 No. 600 “On Approval of the Model Rules for Admission to Education in Educational Organizations Implementing Educational Programs of Higher and Postgraduate Education” (hereinafter referred to as the Model Rules).

2. The entrance exam for doctoral studies consists of writing an essay, passing a test for readiness for doctoral studies (hereinafter referred to as TRDS), an exam in the profile of a group of educational programs and an interview.

<b>Block</b>	<b>Points</b>
1. Essay	10
2. Test for readiness for doctoral studies	30
3. Exam according to the profile of the group of the educational program	40
4. Interview	20
Total admission score	100/75

3. The duration of the entrance exam is 4 hours, during which the applicant writes an essay, passes a test for readiness for doctoral studies, and answers an electronic examination. The interview is conducted on the basis of the university before the entrance exam.

**2. Procedure for the entrance examination.**

1. Applicants for doctoral studies in the group of educational programs D093 - «Mechanics» write a problematic / thematic essay. The volume of the essay is at least 250-300 words.

2. The electronic examination card consists of 3 questions.

## List of examination topics

### Subject «Theoretical mechanics»

1. The subject of theoretical mechanics, basic concepts and definitions. Point and rigid body kinematics. Methods for specifying the movement of a point. Speed and acceleration in curved motion. Expansion of acceleration along the axes of a natural trihedron.
2. Mechanical system. The translational motion of an absolutely rigid body. Rotational movement of an absolutely rigid body around a fixed axis. Angular velocity and angular acceleration. Velocities and accelerations of points when rotating a rigid body.
3. Plane-parallel movement of an absolutely rigid body. Velocities and accelerations of points of a flat figure. Instantaneous centers of speeds and accelerations.
4. The movement of a rigid body about a fixed point. Euler angles. Euler's kinematic equations. Euler - d'Alembert theorem. Velocity and acceleration of points of a body moving about a fixed point.
5. Complex motion of a rigid body. Reduction of the system of sliding vectors. The main vector and the main point. Invariants of reduction of a system of sliding vectors. Screw.
6. Motion of a free rigid body. Chassel's theorem. Velocities and accelerations of points of a free rigid body.
7. Complex movement of a point. Absolute, relative, figurative movement. Velocity addition theorem. Coriolis theorem.
8. Basic definitions and axioms of statics. Moment of force relative to the center. The moment of force about the axis.
9. System of converging forces. Equilibrium conditions for a system of converging forces. Parallel forces system. Equilibrium conditions, equivalent equilibrium conditions. Center of gravity. Methods for finding the center of mass.
10. The theory of pairs. A system of forces arbitrarily located in space. Equilibrium conditions for various systems of forces. Statically undefined systems.
11. Dynamics of a point and a system of material points. Rectilinear oscillations of a point (harmonic, damped, forced). Differential equations of motion for a system of material points.
12. General theorems of the dynamics of a point. Basic dynamic quantities of the system. General theorems of system dynamics.
13. Types of connections. Elementary work of strength. The work of the force of gravity, elastic force, friction force. Basic concepts.
14. Virtual and true movements. Variation of coordinates. The number of degrees of freedom.
15. Generalized coordinates, speeds and forces. Conditions imposed by constraints on coordinate variations. The principle of possible displacements.
16. The d'Alembert principle. General theorems deduced from the d'Alembert principle. D'Alembert-Lagrange principle.
17. Method of Lagrange multipliers. Lagrange equations of the first kind. Holonomic and nonholonomic systems. Determination of reactions using Lagrange equations of the 1st kind.
18. Lagrange equations of the second kind. Lagrange equations for a system under the influence of potential forces. Lagrange function. Integral of energy.
19. Cyclic coordinates. Method of ignoring coordinates. Routh function. Routh's equations. Cyclic integral.
20. Canonical equations. Canonical transformations. Advantages of Canonical Equations.
21. Geometry of the masses. Huygens-Steiner theorem. Moment of inertia about intersecting axes. Tensor and ellipsoid of inertia. Main axes of inertia.
22. Differential equations for the rotational motion of a rigid body. Axle pressure. Plane-parallel movement of an absolutely rigid body.

23. The movement of an absolutely rigid body with one fixed point. Basic dynamic quantities. Koenig's theorems. Euler's dynamic equations.

24. General formulation of the problem of the motion of a heavy rigid body with a fixed point. Differential equations of motion. Particular cases of integration: the cases of Euler, Lagrange, Kovalevskaya.

25. Canonical perturbation theory. Variables Delaunay, Andoillet. Action-angle variables.

### **Subject «Continuum mechanics»**

1. Subject of continuum mechanics, main problems and variety of its applications. Various properties of solid, liquid and gaseous bodies. Continuity hypothesis.

2. Elements of tensor calculus and analysis. Basic differential operations on tensors. Gradient, divergence, rotor, Laplacian.

3. Kinematics of a continuous medium. Equations of motion of particles of a continuous medium. Lagrange and Euler's methods for studying the motion of a continuous medium and their relationship. Scalar and vector fields and their main characteristics. Trajectory, streamline, vortex line and their differential equations. Jet, stream tube, vortex tube.

4. The theory of deformations. Elongation coefficient. Strain tensor. The geometric meaning of its components. Strain tensor invariants. Volumetric expansion coefficient. Deformation compatibility condition. Strain rate tensor. The formula and the Cauchy-Helmholtz theorem.

5. Basic theorem and equation of dynamics of continuous medium. Weight. Density of the medium. The law of conservation of mass. Continuity equation for the Lagrange and Euler variables. Mass and surface forces. Stress tensor. The theorem on the change in the amount of motion of the medium. Dynamics equations in "stresses".

6. Equations of equilibrium of the medium. Theorem on the change in the kinetic moment of the medium. Symmetrical and asymmetric stress tensor. Kinetic energy. Theorem on the change in the kinetic energy of the medium.

7. Classical models of continuous media. Model of an ideal incompressible fluid. Euler's equations. Ideal gas model in a barotropic process. Model of a viscous incompressible fluid. Navier - Stokes equations. Viscous gas model. Complete system of equations.

8. Model of an elastic body. Equations of state for isothermal and adiabatic processes and generalized Hooke's law. Complete system of basic equations of the linear theory of elasticity. Lamé equations. Thermoelastic body model. Hooke's law taking into account temperature stresses. Ideal plastic body model.

### **Subject «Fluid and gas mechanics»**

1. Basics of hydrostatics. Equilibrium equations for liquids and gases. Equilibrium in the field of gravity. Equilibrium of a homogeneous incompressible heavy fluid. Equilibrium of a perfect gas in the field of gravity. Archimedes' law.

2. General theory of motion of ideal liquids and gases. Equations of motion of an ideal medium in the form of Gromeki-Lemba. Bernoulli's theorem and integral. Examples of Bernoulli integral applications.

3. The energy equation for the adiabatic motion of an ideal gas. Enthalpy. The energy integral and its application. The speed of propagation of small perturbations in an ideal gas. Sound speed. Newton's and Laplace's formulas. Mach number.

4. One-dimensional stationary motion of an ideal gas through a pipe of variable cross-section. Elementary theory of the Laval nozzle. An example of a plane stationary shock wave. Hugoniot's equation.

5. Irrotational motion of an ideal environment. Speed potential. Lagrange-Cauchy integral. Plane irrotational motion of an ideal incompressible fluid. Current function. Application of the theorem of the function of complex variables. Comprehensive potential. Examples of the

simplest currents.

6. Dynamics of a viscous incompressible fluid. The Navier-Stokes equation for the dynamics of a viscous fluid in dimensionless variables. Dimensionless parameters and their meaning. Reynolds number.

7. The movement of a viscous incompressible fluid in a round pipe. Poiseuille's law. Examples of the simplest flows at low Reynolds numbers. Features of the flow at large Reynolds numbers. The concept of the boundary layer. Prandtl equations. Blasius problem.

8. Laminar and turbulent movements. Reynolds' experience. Reynolds equation for averaged turbulent motion. Boussinesq's formula. Prandtl's conjecture. Review of other semi-empirical theories of turbulence.

### **Subject «Mechanics of deformable solids»**

1. Properties of isotropy and anisotropy. Cylindrical anisotropy. Spherical anisotropy.

2. The main tasks of the theory of elasticity. Statement of problems of the linear theory of elasticity in stresses and displacements. Lamé and Beltrami-Mitchell equations. Representation of the solution of the Lamé equation in the Popkovich-Neuber and Boussinesq-Galerkin forms. Saint-Venant principle. Voltage function. Thick-walled pipes problem.

3. The Clapeyron equation and the uniqueness theorem for the solution of basic problems in the linear theory of elasticity. Betty's reciprocity theorem. Influence tensor. Maxwell's theorem. Potentials of the theory of elasticity. Determination of the displacement field for given external forces and a vector of displacements on the body surface. Ritz and Bubnov - Galerkin variational methods.

4. Plane problems of the theory of elasticity. Their types. Airy stress function. Complex representation of the displacement vector, stress tensor and biharmonic function. Rigid stamp problem. Hertz's problem on the compression of elastic bodies.

5. Basic relations of the moment theory of elasticity. The effects of moment stresses in the linear theory of elasticity. Fundamentals of the theory of magnetoelasticity and thermoelasticity. Basic concepts of thermoviscoelasticity. Strength conditions. Long lasting durability. The laws of state of a nonlinear elastic body. Representation of the law of state by a quadratic trinomial. Murnaghan's law of state. Statement of problems and main results of the theory of elastic waves.

6. Model of a perfectly plastic body. Loading and yield surfaces. Residual plastic deformations. The simplest concrete models. Concepts of simple and complex loading. Plasticity conditions.

7. The laws of the formation of plastic deformations. Associated Law. Flow theory. Deformation theory of plasticity. Method of elastic solutions. Model of a plastic medium with hardening.

8. Plane problems of the theory of plasticity. Sliding lines. Basic properties of slip lines. The problem of torsion of rods with plastic regions.

9. The postulate of stability and its applications in the theory of plasticity and creep of materials. Models of complex environments.

10. Strength and destruction. Classical theories of strength. Cracked body model. Destruction criteria. Crack mechanics. Scattered fracture mechanics.

### **Literature**

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